Math 72 7.5 Root and Power Functions

Objectives

1) Evaluate root and power functions
2) Graph root and power functions
3) Find the domain of root and power functions

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7.6 Equations involving Radical Expressions

Objectives

1) Solve radical equations containing one radical
2) Reject extraneous solutions for equations with even-index radicals
3) Solve radical equations containing two square roots.
4) Use the Pythagorean Theorem
5) Use the Distance Formula
6) Solving equations containing powers

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A **root function** is a function $f(x) = x^{1/n} = \sqrt[n]{x}$.

A **power function** is a function $f(x) = x^{m/n} = \sqrt[n]{x^m}$

⇒ So a root function is a special case of a power function;
A root function is also a power function.

⇒ If $n = 1$, a power function does not have a radical.

Write each function as a radical.

NO ① $f(x) = x^{1/3}$

$\boxed{f(x) = \sqrt[3]{x}}$

NO ② $f(x) = x^{3/4}$

$\boxed{f(x) = \sqrt[4]{x^3}}$

③ $f(x) = x^{-2/5}$

YES

$f(x) = \dfrac{1}{x^{2/5}} = \boxed{\dfrac{1}{\sqrt[5]{x^2}} = f(x)}$

Evaluate each function. Round to nearest thousandth if necessary.

④ $f(x) = \sqrt[4]{x+1}$ $\quad x = -15, \quad x = 15$

NO

$f(-15) = \sqrt[4]{-15+1} = \sqrt[4]{-14} = \boxed{\text{not real}}$

$f(15) = \sqrt[4]{15+1} = \sqrt[4]{16} = \sqrt[4]{2^4} = \boxed{2}$

NO ⑤ $f(x) = \sqrt[3]{(x+1)^2}$ $\quad x=-2, \quad x=7$

$f(-2) = \sqrt[3]{(-2+1)^2}$

$= \sqrt[3]{(-1)^2}$

$= \sqrt[3]{1}$

$= \boxed{1}$

$f(7) = \sqrt[3]{(7+1)^2}$

$= \sqrt[3]{8^2}$ $\quad$ or $\quad = \sqrt[3]{64}$

$= (\sqrt[3]{8})^2$ $\quad = \boxed{4}$

$= 2^2$

$= \boxed{4}$

NO ⑥ $f(x) = x^{4/3}$ $\quad x = -8, \quad x = 27$

$f(-8) = (-8)^{4/3}$

$= \sqrt[3]{(-8)^4}$ $\quad$ or $\quad \sqrt[3]{4096}$

$= (-2)^4$ $\quad = \sqrt[3]{16^3}$

$= \boxed{16}$ $\quad = \boxed{16}$

$f(27) = (27)^{4/3}$

$= (\sqrt[3]{27})^4$ $\quad$ or $\quad \sqrt[3]{27^4}$

$= 3^4$ $\quad = \sqrt[3]{531441}$

$= \boxed{81}$ $\quad = 81$

NO ⑦ $f(x) = x^{-3/4}$ $\quad x = 16, \quad x = -3$

$f(16) = 16^{-3/4}$

$= \dfrac{1}{16^{3/4}}$

$= \dfrac{1}{(\sqrt[4]{16})^3} = \dfrac{1}{2^3} = \boxed{\dfrac{1}{8}}$

1.6

Warm up: Simplify

No ① $(\sqrt{2})^2$

$= \sqrt{2} \cdot \sqrt{2}$

$= \boxed{2}$

No ② $(\sqrt{x})^2$

$= \sqrt{x} \cdot \sqrt{x}$ ← So long as $x > 0$ we can use the product rule for radicals.

$= \boxed{x}$

No ③ $(\sqrt{2} + \sqrt{3})^2$

$= (\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})$

$= 2 + \sqrt{6} + \sqrt{6} + 3$

$= \boxed{5 + 2\sqrt{6}}$

yes ④ $(\sqrt{2x} + \sqrt{3})^2$

$= (\sqrt{2x} + \sqrt{3})(\sqrt{2x} + \sqrt{3})$ ← So long as $2x > 0$ (so $x > 0$) we can use the product rule for radicals

$= \boxed{2x + 2\sqrt{6x} + 3}$

yes ⑤ $(\sqrt{2x+3})^2$

$= \sqrt{2x+3} \cdot \sqrt{2x+3}$

$= \boxed{2x+3}$ ← So long as $2x + 3 \geq 0 \Rightarrow x \geq -\frac{3}{2}$ we can use the product rule for radicals

no ⑥ Solve $\dfrac{2}{a-1} + \dfrac{3}{a+1} = \dfrac{-6}{a^2-1}$

Remember: $a \neq 1$ and $a \neq -1$

If we get $a = 1$ or $a = -1$, we reject them as extraneous solutions.

NO ③ $\frac{2}{a-1} + \frac{3}{a+1} = \frac{-6}{(a+1)(a-1)}$

mult all by LCD $= (a+1)(a-1)$

$$\frac{2}{\cancel{(a-1)}}(a+1)\cancel{(a-1)} + \frac{3}{\cancel{(a+1)}}\cancel{(a+1)}(a-1) = \frac{-6}{\cancel{(a+1)}\cancel{(a-1)}}\cancel{(a+1)}\cancel{(a-1)}$$

$2(a+1) + 3(a-1) = -6$

$2a+2+3a-3=-6$

$5a-1=-6$

$5a=-5$

$a=1$ reject

$\boxed{\emptyset}$

Solving a rational equation is not related to today's lesson — but radical equations can also have extraneous solutions.

NO ④ Solve $\sqrt{2x-3} - 5 = 0$

step 1: Isolate the radical. (add 5 both sides)

$\sqrt{2x-3} = 5$

step 2: Square entire LHS, entire RHS, using parentheses if two or more terms/signs

$(\sqrt{2x-3})^2 = (5)^2$

$2x-3=25$

step 3 : Isolate the variable

$$\frac{2x}{2} = \frac{28}{2}$$

$$x = 14$$

step 4: Not optional! Plug in to check if answer is extraneous. Must use original equation, before we squared both sides.

$$\sqrt{2x-3} - 5 \stackrel{?}{=} 0$$

$$\sqrt{2(14)-3} - 5 = 0$$

$$\sqrt{28-3} - 5 = 0$$

$$\sqrt{25} - 5 = 0$$

$$5 - 5 = 0 \checkmark$$

$\boxed{x = 14}$ is a valid solution.

No ⑤ Solve $\sqrt{5x+6} - 3 = -2$

isolate radical

$$\sqrt{5x+6} = 1$$

$$(\sqrt{5x+6})^2 = 1^2$$

$$5x + 6 = 1$$

$$5x = -5$$

$$x = -1$$

check $\sqrt{5(-1)+6} - 3 = -2$

$$\sqrt{-5+6} - 3 = -2$$

$$\sqrt{1} - 3 = -2$$

$$1 - 3 = -2 \checkmark$$

$\boxed{x = -1}$

YES ⑥ Solve $\sqrt{3x-5}+8=3$

$$\sqrt{3x-5}=-5$$

isolate radical

Wide awake? Remember that $\sqrt{\ }$ means the principle square root, which is positive.

$\Rightarrow$ $\boxed{\text{no solution}}$

$$\left(\sqrt{3x-5}\right)^2=(-5)^2$$

Sound asleep?
Square both sides
Notice $(-5)^2$ was negative, but becomes positive.
This is how extraneous solutions are created.

$$3x-5=25$$
$$3x=30$$
$$x=10$$

check $\sqrt{3(10)-5}+8 \overset{?}{=} 3$

$$\sqrt{30-5}+8=3$$
$$\sqrt{25}+8=3$$
$$5+8\neq 3$$
$$13\neq 3$$

reject! $\boxed{\text{no solution}}$

VES (7) Solve $\sqrt{3x-11} = x-5$

step 1: radical is already isolated.

step 2: square both sides, using parentheses

$$\left(\sqrt{3x-11}\right)^2 = (x-5)^2$$

IMPORTANT

$(x-5)^2 \neq x^2 - 5^2$

$(x-5)^2 = (x-5)(x-5)$

$= x^2 - 10x + 25$

Step 3: Foil to simplify RHS

$$3x - 11 = x^2 - 10x + 25$$

step 4: Notice the type of equation. This is $x^2$, quadratic.

To solve a quadratic

1) set $= 0$.
2) factor completely
3) set factors $= 0$.

$$0 = x^2 - 10x + 25 - 3x + 11$$

$$x^2 - 13x + 36 = 0$$

$$(x-9)(x-4) = 0$$

36

$-9$ $-4$

$-13$

$$x = 9, 4$$

step 5 Check both answers in original equation.

$x = 9$: $\sqrt{3(9)-11} = 9 - 5$

substitute $x=9$ in both locations

$$\sqrt{27-11} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4$$

Keep $x = 9$.

$x=4 \qquad \sqrt{3(4)-11} = 4-5$

$$\sqrt{12-11} = -1$$

$$\sqrt{1} = -1$$

$$1 \neq -1 \qquad \text{reject } x=4.$$

$\boxed{x=9}$ is the only valid solution.

YES (8) Solve $\sqrt[3]{p^2-4p-4} = \sqrt[3]{-3p+2}$

> Notice: cube roots, index 3.
> This means two important things:
> 1) We will raise both sides to the 3rd power ("cube both sides") instead of 2nd power.
> 2) $\sqrt[3]{\text{neg}}$ is a valid result, so we will not have to check for extraneous results.

step1: cube both sides

$$\left(\sqrt[3]{p^2-4p-4}\right)^3 = \left(\sqrt[3]{-3p+2}\right)^3$$

$$p^2-4p-4 = -3p+2$$

step2: Notice $p^2 \Rightarrow$ quadratic. Set $=0$, factor

$$p^2-p-6=0$$

$-6$

$-3 \quad +2$

$-1$

$$(p-3)(p+2)=0$$

$$\boxed{p=3 \qquad p=-2}$$

> In general: If the index is even, we must check for extraneous solutions. If the index is odd, checking is optional and only to check our work.

NO

(9) Solve $\sqrt{z^2 - z - 7} + 3 = z + 2$

Step 1: Isolate the radical

$$\sqrt{z^2 - z - 7} = z - 1$$

Step 2: Square both sides using parentheses.

$$\left(\sqrt{z^2 - z - 7}\right)^2 = (z-1)^2$$

Step 3: Foil RHS

$$z^2 - z - 7 = (z-1)(z-1)$$
$$= z^2 - z - z + 1$$
$$= z^2 - 2z + 1$$

$$z^2 - z - 7 = z^2 - 2z + 1$$

Step 4: Subtract $z^2$ both sides, then isolate z

$$-z - 7 = -2z + 1$$
$$+2z \qquad\quad +2z$$

$$z - 7 = 1$$
$$+7 \quad +7$$
$$z = 8$$

Step 5: check for extraneous by subst into original.

$$\sqrt{8^2 - 8 - 7} + 3 = 8 + 2$$

plug in 3 locations

$$\sqrt{64 - 8 - 7} + 3 = 10$$
$$\sqrt{56 - 7} + 3 = 10$$
$$\sqrt{49} + 3 = 10$$
$$7 + 3 = 10 \checkmark$$

$$\boxed{z = 8}$$

YES (10) Solve $\sqrt{x+5} - \sqrt{x} = 1$

Notice
There are two radicals.
We will square twice!

DON'T DO THIS!!

Remember: FOIL

$(\sqrt{x+5} - \sqrt{x})^2$

$= (\sqrt{x+5} - \sqrt{x})(\sqrt{x+5} - \sqrt{x})$

$= x+5 - \sqrt{x(x+5)} - \sqrt{x(x+5)} + x$

$= 2x+5 - 2\sqrt{x(x+5)}$

still have a radical ☹
+
it's even uglier ☹

step 1: Isolate one of the radicals.

$$\sqrt{x+5} = \sqrt{x} + 1$$

step 2: Square both sides, using parentheses

$$(\sqrt{x+5})^2 = (\sqrt{x} + 1)^2$$

$$x+5 = (\sqrt{x} + 1)(\sqrt{x} + 1) \qquad \text{FOIL}$$

$$x+5 = x + \sqrt{x} + \sqrt{x} + 1$$

$$x+5 = x + 1 + 2\sqrt{x} \qquad \text{Combine}$$

still have a radical!

Step 3: Isolate the remaining radical, or the radical and its coefficient.

$$\begin{array}{rcl} x + 5 & = & x + 1 + 2\sqrt{x} \\ -x & & -x \\ \hline 5 & = & 1 + 2\sqrt{x} \\ -1 & & -1 \\ \hline 4 & = & 2\sqrt{x} \end{array}$$

$\frac{2\sqrt{x}}{2} = \frac{4}{2}$ swap LHS and RHS

$\sqrt{x} = 2$ isolate $\sqrt{x}$

$(\sqrt{x})^2 = (2)^2$ square both sides

$x = 4$

$\sqrt{x+5} - \sqrt{x} = 1$

$\sqrt{4+5} - \sqrt{4} = 1$ check for extraneous

$\sqrt{9} - \sqrt{4} = 1$

$3 - 2 = 1$ ✓

$\boxed{x=4}$

yes

(11) Solve $\sqrt{2x+6} - \sqrt{x-1} = 2$

$\sqrt{2x+6} = \sqrt{x-1} + 2$ Isolate a radical.

$(\sqrt{2x+6})^2 = (\sqrt{x-1} + 2)^2$ square both sides, using parentheses

$2x+6 = (\sqrt{x-1} + 2)(\sqrt{x-1} + 2)$ FOIL RHS

$2x+6 = x-1 + 2\sqrt{x-1} + 2\sqrt{x-1} + 4$

$2x+6 = x+3 + 4\sqrt{x-1}$ combine

Still have one radical

$4\sqrt{x-1} + x + 3 = 2x + 6$ swap LHS + RHS

$\phantom{4\sqrt{x-1}} -x \quad -3 \quad\; -x \quad -3$

$4\sqrt{x-1} = x + 3$ isolate the radical with its coefficient.

$(4\sqrt{x-1})^2 = (x+3)$ square both sides, using parentheses

$16(x-1) = (x+3)(x+3)$ FOIL RHS, distribute LHS

$16x - 16 = x^2 + 3x + 3x + 9$ combine like terms

Notice quadratic

$$\begin{array}{rcl} 16x - 16 & = & x^2 + 6x + 9 \\ \underline{-16x \quad +16} & & \underline{\quad -16x \quad +16} \end{array}$$

set $= 0$.

$0 = x^2 - 10x + 25$

$x^2 - 10x + 25 = 0$ swap LHS and RHS

$(x-5)(x-5) = 0$ factor completely

$x - 5 = 0 \quad x - 5 = 0$ set factors $= 0$

$x = 5$

check in original equation

$\sqrt{2(5)+6} - \sqrt{5-1} = 2$

$\sqrt{10+6} - \sqrt{4} = 2$

$\sqrt{16} - \sqrt{4} = 2$

$4 - 2 = 2$ ✓

$\boxed{x = 5}$

yes. (12) Solve $r = \sqrt{\dfrac{S}{4\pi}}$ for S.

square both sides to remove radical

$r^2 = \left(\sqrt{\dfrac{S}{4\pi}}\right)^2$

$r^2 = \dfrac{S}{4\pi}$

isolate S by mult by $4\pi$

$\boxed{4\pi r^2 = S}$

NO

(13) Solve $(4x+1)^{1/2} = 5$

Recall: exponent ½ means square root

$$\sqrt{4x+1} = 5$$

square both sides

$$\left(\sqrt{4x+1}\right)^2 = 5^2$$

$$4x+1 = 25$$
$$-1 \quad -1$$

$$\frac{4x}{4} = \frac{24}{4}$$

$$x = 6$$

check $\sqrt{4(6)+1} = 5$

$$\sqrt{25} = 5 \checkmark$$

$$\boxed{x=6}$$

$$\left((x-2)^{3/2}\right)^{2/3} = 8^{2/3}$$

step 3: multiply exponents (exponent law) to get 1.

$$(x-2)^1 = 8^{2/3}$$
$$x-2 = 8^{2/3}$$

step 4: write $^{2/3}$ using radicals and simplify

$$x-2 = \sqrt[3]{8^2} = (\sqrt[3]{8})^2$$
$$= \sqrt[3]{64} \quad | = (2)^2$$
$$= 4 \quad | = 4$$

step 5: Isolate $x$

$$x-2=4$$
$$+2 \quad +2$$
$$x=6$$

step 6: check for extraneous

$$(x-2)^{3/2} = 8$$
$$(\sqrt{6-2})^3 = 8$$
$$(\sqrt{4})^3 = 8$$
$$2^3 = 8 \checkmark$$
$$\boxed{x=6}$$

No 15 Suppose $f(x) = \sqrt{x-2}$

Solve $f(x) = 1$. What point is on the graph.

step 1: replace $f(x)$ by 1

$$1 = \sqrt{x-2}$$

step 2: square both sides

$$1^2 = (\sqrt{x-2})^2$$

$1 = x - 2$
$+2 \quad +2$

$\boxed{3 = x}$

check: (because denom of original exp is even.)

$\sqrt{3-2} = 1$ ✓

$x=3$, $y=1$ means ordered pair $\boxed{(3,1)}$ is on the graph

NO (16) A plural birth is a live birth to twins, triplets, or more. The function $R(t) = 26 \cdot \sqrt[10]{t}$ models the plural birth rate (R plural births per 1000 live births) where t is the number of years since 1995.

a) Use the model to predict the year in which the plural birth rate will be 39. Round to the nearest year.

Subst $R=39$

$$39 = 26 \sqrt[10]{t}$$

Isolate radical

$$\frac{39}{26} = \sqrt[10]{t}$$

Reduce fraction by $\div 13$

$$\frac{3}{2} = \sqrt[10]{t}$$

Raise both sides to 10th power

$$\left(\frac{3}{2}\right)^{10} = \left(\sqrt[10]{t}\right)^{10}$$

$$\frac{3^{10}}{2^{10}} = t$$

$$\frac{59049}{1024} = t$$

Calculate and round $57.66 \Rightarrow 58$ years after 1995

Add 58 to 1995 $= \boxed{2053}$

b) Use the model to predict the year in which the birth rate will be 36. Round to the nearest year.

$$36 = 26\sqrt[10]{t}$$

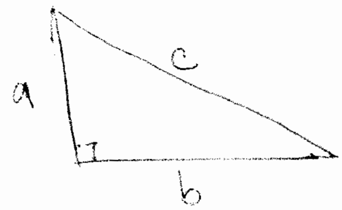
$$\frac{36}{26} = \sqrt[10]{t}$$

$$\left(\frac{18}{13}\right)^{10} = \left(\sqrt[10]{t}\right)^{10}$$

$$t = \frac{195101}{7533} \approx 25.89 \longrightarrow 26 \text{ yrs}$$

$$\begin{array}{r} 1995 \\ +\ 26 \\ \hline \boxed{2021} \end{array}$$

The Pythagorean Theorem can be applied when problem has side lengths (not angles) of a right triangle.

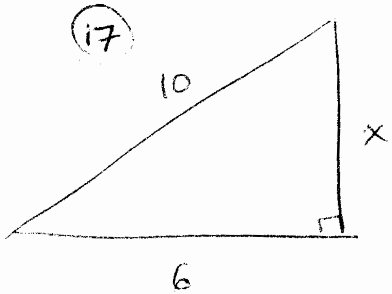



c must be the hypotenuse across from the right angle.

a and b are the legs of the right triangle.

Find the length of the missing side of the right triangle

17

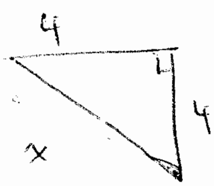



$$a^2 + b^2 = c^2$$
$$6^2 + x^2 = 10^2$$
$$36 + x^2 = 100$$
$$x^2 = 64$$
$$x = \pm\sqrt{64}$$
$$x = \pm 8$$

but $x = -8$ is extraneous

$$\boxed{x = 8}$$

18

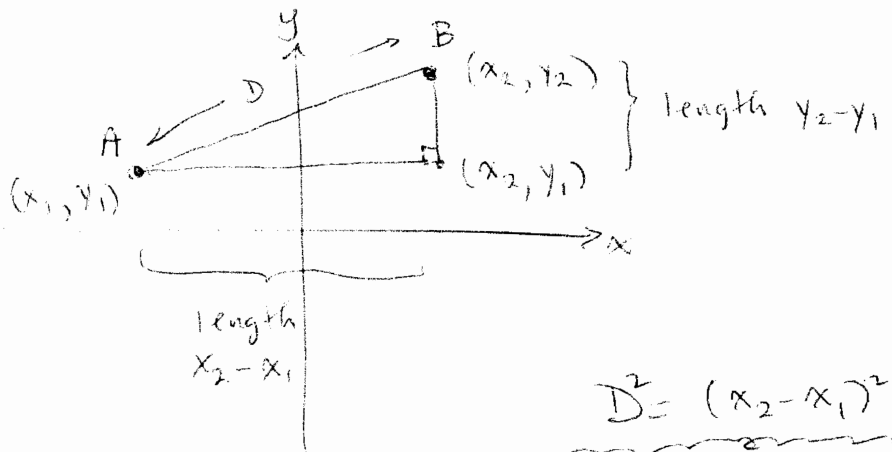



$$a^2 + b^2 = c^2$$
$$4^2 + 4^2 = x^2$$
$$16 + 16 = x^2$$
$$32 = x^2$$
$$\pm\sqrt{32} = x$$

reject negative as extraneous

$$x = \sqrt{16 \cdot 2}$$
$$\boxed{x = 4\sqrt{2}}$$

The Distance Formula for points on the x-y plane is an application of the Pythagorean Theorem.




$$D^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

so

$$\boxed{D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$ *Memorize!

Distance Formula

(19) Find the distance between $(3, 9)$ and $(-4, 2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4-3)^2 + (2-9)^2}$$

$$= \sqrt{(-7)^2 + (-7)^2}$$

$$= \sqrt{49 + 49}$$

$$= \sqrt{98}$$

$$= \sqrt{7^2 \cdot 2}$$

$$= \boxed{7\sqrt{2}}$$

98

2 49

7 7

(20) Find $x$ if the distance is $d$

$(x, 3)$ and $(12, -4)$ $\quad d = 25$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$25 = \sqrt{(12 - x)^2 + (-4 - 3)^2}$$

$$25 = \sqrt{(12 - x)^2 + (-7)^2}$$

square both sides
to clear radical

$$(25)^2 = \left(\sqrt{(12 - x)^2 + 49}\right)^2$$

$$625 = (12 - x)^2 + 49$$

$$576 = (12 - x)^2$$

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Option 1: Square root property

$$\pm\sqrt{576} = 12 - x$$

$$\pm 24 = 12 - x$$

| | |
|---|---|
| $12 - x = 24$ | $12 - x = -24$ |
| $-x = 12$ | $-x = -36$ |
| $\boxed{x = -12}$ | $\boxed{x = 36}$ |

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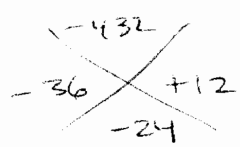
Option 2: FOIL, set = 0, factor

$$576 = (12 - x)(12 - x)$$

$$576 = 144 - 24x + x^2$$

$$0 = x^2 - 24x - 432$$




$$0 = (x - 36)(x + 12)$$

$\boxed{x = 36}$ $\boxed{x = -12}$

Why is a negative result okay? Because $x$ represented an $x$-coordinate on the plane.